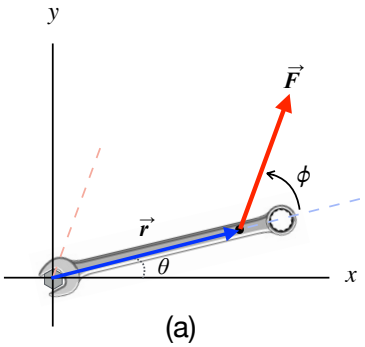
Calculating Torque

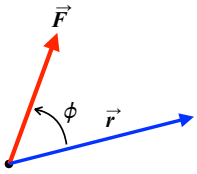
CONCEPT: Torque ("moment" in engineering) is the rotational counterpart of force. It is a measure of the ability of a force to cause or change the rotation of an object.

The torque $\vec{\tau}$ about a point P, called the pivot or fulcrum, is given by the cross product between the position vector $\vec{r}$ and the force vector $\vec{F}$.

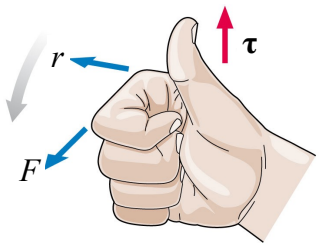
$$\vec{\tau} = \vec{r} \times \vec{F}$$



(a)



(b)



(c)

MAGNITUDE: The magnitude of the torque about a pivot point is given by:

$$\tau = r\,F \sin\phi$$

where:

- r is the magnitude of the position vector $\vec{r}$,
  it is the distance from the pivot to the point of application of the force (see Fig. a);
- F is the magnitude of the force; and
- ϕ is the angle between the tails of $\vec{r}$ and $\vec{F}$ (see Fig. b).

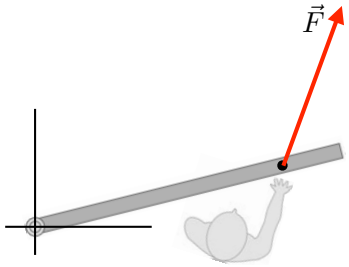
DIRECTION: The torque vector is perpendicular to the plane defined by vectors $\vec{r}$ and $\vec{F}$. In the example above, the torque points along the +z-axis, out of the page.

Use the right-hand rule to determine direction (see Fig. c). By convention, the direction of the torque is considered:

- ⊕ **positive** (+z-axis) if it causes a counterclockwise (ccw) rotation; and
- ⊖ **negative** (–z-axis) if it causes a clockwise (cw) rotation.

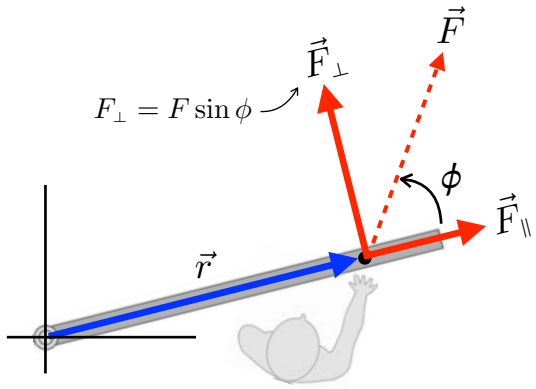
VISUALIZING TORQUE:
Torque requires a perpendicular component between the vectors $\vec{r}$ and $\vec{F}$.

There are two ways of defining it:
1. due to the component of the force perpendicular to $\vec{r}$
2. due to the component of $\vec{r}$ perpendicular to the force.



top view of a person pushing a door, producing a torque about the hinge

Method 1

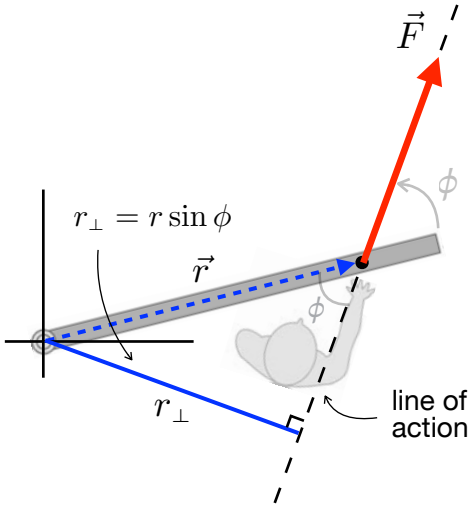


- identify the vector $\vec{r}$, from the pivot to the point of application of the force;
- identify $\vec{F}_\perp$, the component of the force perpendicular to $\vec{r}$ (while $\vec{F}_\parallel$ is the component parallel to $\vec{r}$).

toque due to distance *r* multiplied by the perpendicular component of the force:

$$\tau = r\,(F_\perp)$$

Method 2



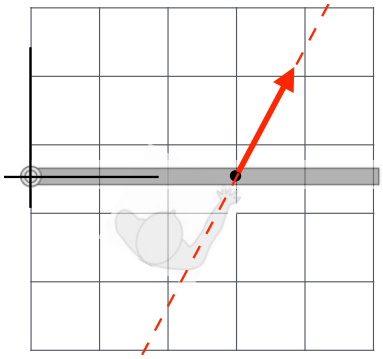
- identify the line of action, the imaginary line along which the force acts;
- identify the moment or lever arm $r_\perp$, which is the minimum distance from the pivot to the line of action.

torque due to moment arm $r_\perp$ multiplied by the force:

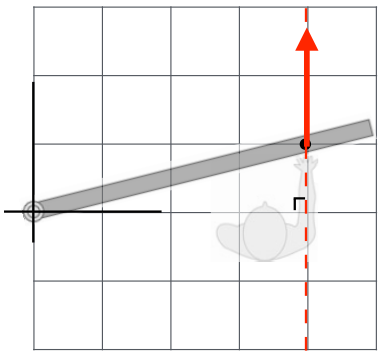
$$\tau = (r_\perp)\,F$$

Note:

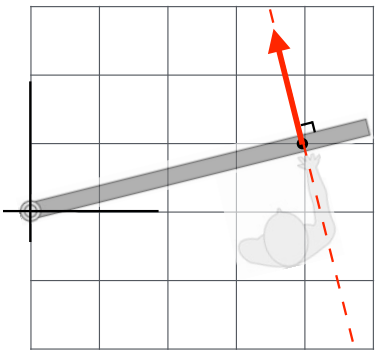
- Both methods of visualizing torque are equivalent and provide the same result.
- Select a method based on whether you know r or $r_\perp$.
- Recall that when the line of action of a force crosses the pivot, the torque exerted by that force is zero.



r is known, *r* = 3 units



$r_\perp$ is known, $r_\perp$ = 4 units



$r = r_\perp$